ECONOMICS 214: INTERMEDIATE MACROECONOMICS
MATH PRIMER.

In the Intermediate Macroeconomics course we will use a bit of calculus (derivatives) and algebra. This primer is intended to freshen your memory and to prepare you for the level of math in this course.

- **Derivatives**

Let's say a function Y is of the following form: Y=K^β
and we want to derive the effects on Y when K changes.
The (partial) derivative, $\frac{\partial Y}{\partial K}$, is derived as follows:

Multiply the function with the K exponent and then subtract 1 from the exponent: $\frac{\partial Y}{\partial K} = \beta K^{\beta-1}$

This derivative can also be written as $\frac{\beta K^\beta}{K}$

since $K^{\beta-1} = K^\beta K^{-1} = \frac{K^\beta}{K}$ (inverse of K, K^{-1}, is the same as $\frac{1}{K}$)

The derivative shows how Y changes when K changes by 1 unit.

Other examples:

Function:	Derivatives:
Y=AK^β	$\frac{\partial Y}{\partial K} = A\beta K^{\beta-1}$ and $\frac{\partial Y}{\partial A} = K^\beta$
Y=$AK^B L^\alpha$	$\frac{\partial Y}{\partial A} = K^\beta L^\alpha$ and $\frac{\partial Y}{\partial K} = A\beta K^{\beta-1} L^\alpha$ and $\frac{\partial Y}{\partial L} = AK^\beta \alpha L^{\alpha-1}$
C = a + bY	$\frac{\partial C}{\partial Y} = b$

Partial vs. Total derivatives

Lower case delta (∂) implies a partial derivative and upper case delta (Δ) implies a total derivative. The difference lies in what assumptions we make regarding other variables in the function. The partial derivative, $\partial Y/\partial K$, shows how Y changes when K changes (by 1 unit) holding all other variables unchanged (or constant). The total derivative includes changes in other variables in the function. Here is an example.

Y=AK^β The partial derivatives would be as in the previous section.

The total derivative would be $\frac{\Delta Y}{\Delta K} = A\beta K^{\beta-1} + \frac{\Delta A}{\Delta K} K^\beta$

The total derivatives are a bit more complicated since they have more terms than the partial derivatives. However, there is no need to worry since we only use partial derivatives in this course.

- **Algebra**

When we derive multipliers we need to make a distinction between endogenous and exogenous variables. Endogenous variables have an explicit function (like Y and C below) whereas the exogenous variables are assumed to be given (no equation for them).

For example, let's say we have the following national income identity (Y=output, C=consumption, I=investment and G=government expenditure):
$Y=C+I+G$
and the following consumption function (C_0=autonomous consumption and C_Y=marginal propensity to consume):
$C=C_0+C_Y*Y$
In this example, Y and C are endogenous and C_O, C_Y, I and G are assumed to be exogenous. Note that when Y increases (as a result of Co, I or G), C increases which increases Y which increases C ...etc.. The idea is to isolate the endogenous variable (in this case, Y) on one side of the equal sign with all the exogenous variables on the other side:

1. Substitute the C function into the identity: $Y = C_0+C_Y*Y + I + G$
2. Collect (or isolate) the Y terms: $Y-C_Y*Y = C_0 + I + G$
3. Simplify: $Y = \frac{C_0 + I + G}{1 - C_Y}$

Note that we have the endogenous variable Y on the left hand side of the equation and all the exogenous variables on the right hand side. We are now ready to take derivative of, say, the effects on Y when G changes: $\frac{\partial Y}{\partial G} = \frac{1}{1 - C_Y}$
This should be familiar. This is the government expenditure multiplier from the principles of macro course.