- exponents of L & K are income shares of L & K if C.R.S. & Perf. Competition in Factor Markets

$$Y = K^{\alpha} L^{1-\alpha}$$

$$\rightarrow MP_K = \alpha K^{\alpha-1} L^{1-\alpha} = \frac{\alpha[K^{\alpha} L^{1-\alpha}]}{K} = \alpha \frac{Y}{K} \quad (1)$$

$$\rightarrow MP_L = (1-\alpha)\frac{Y}{K} \quad (2)$$

*IF Factors markets perf. competitive. We know factors get paid their Marginal products: $MP_K = R/P$ & $MP_L = w/P$

$$MP_K = \boxed{\frac{R}{P} = \frac{\alpha Y}{K}} \rightarrow \frac{R \times K}{P \times Y} = \alpha$$

($R \times K$: capital income; $P \times Y$: Total income; α = Fraction of Total income paid to K owners)

&

$$\frac{w}{P} = (1-\alpha)\frac{Y}{L} \rightarrow \frac{w \times L}{P \times Y} = (1-\alpha)$$

($w \times L$: Labor income; fraction of total income paid to workers)

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2 arithmetic tricks (P. 26)

1. • Percentage change in $(P \times Y)$ $\approx$ Perc. change in P + Perc. change in Y

(ex) Year 1: P = 2 & Y = 100 → P×Y = 200

Year 2: P = 2.1 & Y = 103 → P×Y = 216.3 ↗ 8.15%

%ΔP = 5% + %ΔY = 3% = 8% ≈

2. • Percent change in (Y/L) $\approx$ Perc. change − Perc. change in L

(ex) Year 1: Y = 100,000 & L = 100 → Y/L = 1,000

Year 2: Y = 110,000 L = 103 → $\frac{Y}{L}$ = 1068 ↗ 6.8%

10% in Y − 3% change in L = 7% ≈

ALSO

- $\Delta \ln Y \approx$ Percent change in Y

(ex) $Y = K^{\alpha} L^{1-\alpha}$: • take ln: $\ln Y = \alpha \ln K + (1-\alpha)\ln L$

• First Δ : $\Delta \ln Y = \alpha \Delta \ln K + (1-\alpha)\Delta \ln L$

or

$\approx$ %ΔY = α(%ΔK) + (1−α)(%ΔL)

(ex) $X = (P \times Y)$

$\ln X = \ln P + \ln Y$

$\Delta \ln X = \Delta \ln P + \Delta \ln Y$

$\approx$ %ΔX = %ΔP + %ΔY.