ECON 214: INTERMEDIATE MACROECONOMICS
IS/LM MODEL ALGEBRAICALLY

We start with a simple model for a closed economy (no NX). We have 3 markets, output market, money market and a bond market. Walras law states that if we have three markets and two of them are (simultaneously) in equilibrium, the third market must also be in equilibrium. Thus, we only need to analyze two of those three markets; We'll focus on the output market and the money market.

Equilibrium in those two markets is defined as follows:

Output market: Supply of goods & services = Demand for goods and services

Money market: Supply of money = Demand for money.

We derive the IS curve from the output market and the LM curve from the money market.

- **Output Market and the IS curve.**

$Y = C + I + G$	: National Income Identity	(1)
$C = C_0 + C_Y(Y-T)$	: Consumption function	(2)
$I = I_0 - I_r r$	: Investment function	(3)
$T = T_0$	: Taxes	(4)
$G = G_0$	: Government expenditure	(5)

C_O = autonomous consumption: Effects of, for example, consumer confidence, wealth (stock market), expectations of future income and prices, uncertainty about future income and any other factors that determine consumption not represented directly in the consumption function.

C_Y = Marginal propensity to consume (MPC).

I_0 = autonomous investment: Effects of, for example, expectations of future profitability, investment tax credits, uncertainty about future economic activity and any other factors that determine investment not represented directly in the investment function.

I_r = Response of investment to changes in the real interest rate.

Substitute (4) into (2):

$$C=C_0 + C_y(Y-T_0) \qquad (6)$$

Substitute (6), (3) and (5) into (1)

$$Y=C_0 + C_Y(Y-T_0) + I_0 - I_r r + G_0 \qquad (7)$$

or

$$Y(1-C_y)=C_0 -C_Y T_0 + I_0 - I_r r+G_0 \qquad (8)$$

Recall that in the IS-LM graph we have the real interest rate (r) on the vertical axis and output (Y) on the horizontal axis, thus, we need to solve (8) in terms of r, i.e. r =....

$$r = \left(\frac{1}{I_r}\right)[C_0 -C_Y T_0+ I_0 + G_0 -Y(1-C_y)] \qquad (9)$$

or, simplifying

$$r = -\left(\frac{1-C_Y}{I_r}\right)Y + \left(\frac{1}{I_r}\right)[C_0 -C_Y T_0+ I_0 + G_0] \qquad \textbf{(10)}$$ **This is the IS equation**.

Defining $\beta_{IS}=\left(\frac{1-C_Y}{I_r}\right)$ and $\alpha_{IS}= \left(\frac{1}{I_r}\right)[C_0 -C_Y T_0+ I_0 + G_0]$, the IS equation can be written as:

$$\mathbf{r} = \alpha_{IS} - \beta_{IS}\mathbf{Y} \qquad \textbf{(11)}$$

Note that the slope of the IS function is negative. Also note that changes in C_0, T_0, I_0 or G_0 changes the intercept (α_{IS}) of the IS function, which means the IS curve shifts.

- **<u>Money Market and the LM curve</u>**

$(M/P)^D = L_o - L_i i + L_y Y$: Real Money Demand (12)

L_0 = Autonomous money demand.

L_i = Response of money demand to changes in the nominal interest rate.

L_y = response of money demand to changes in income.

$(M/P)^S = (M/P)$: Real Money Supply (13)

Set real money supply equal to real money demand:

$(M/P) = L_o - L_i i + L_y Y$ (14)

and solve for i:

$i = \left(\frac{1}{L_i}\right)[L_o + L_y Y - (M/P)]$ **(15) This is the LM equation.**

Defining $\beta_{LM} = \left(\frac{L_y}{L_i}\right)$ and $\alpha_{LM} = \left(\frac{1}{L_i}\right)[L_0 - (M/P)]$, the LM equation can be written as:

$\mathbf{i} = \alpha_{LM} + \beta_{LM}\mathbf{Y}$ (16)

Recall the fisher equation: $i = r + \pi^e$. If expected inflation (π^e) is zero, the nominal interest rate equals the real rate: i=r, which means (16) can be written as:

$\mathbf{r} = \alpha_{LM} + \beta_{LM}\mathbf{Y}$ **(17)**

Note that the slope of the LM function is positive. Also note that changes in L_0 and (M/P) shift the LM curve.

- **<u>Simultaneous equilibrium in the goods and the money markets: Intersection of IS and LM</u>**

When the IS and LM curves intersect, it means <u>both</u> the money and the output markets are in equilibrium. To solve for equilibrium income (Y^*), set IS equation (11) = LM equation (17):

$\alpha_{IS} - \beta_{IS} Y = \alpha_{LM} + \beta_{LM} Y$ (18)

and solve for Y:

$\mathbf{Y}^* = \frac{\alpha_{IS} - \alpha_{LM}}{\beta_{IS} + \beta_{LM}}$ (19) **Equilibrium income.**

To solve for the equilibrium real interest rate (r^*), substitute (19) into the IS (11) or LM (17) equation. Here we substitute into (17):

$\mathbf{r}^* = \alpha_{LM} + \beta_{LM}\left(\frac{\alpha_{IS} - \alpha_{LM}}{\beta_{IS} + \beta_{LM}}\right)$ (20)

or

$$\mathbf{r}^* = \frac{(\beta_{IS} + \beta_{LM})\alpha_{LM} + \beta_{LM}\alpha_{IS} - \beta_{LM}\alpha_{LM}}{\beta_{IS} + \beta_{LM}}$$

which reduces to: $\mathbf{r}^* = \frac{\beta_{IS}\alpha_{LM} + \beta_{LM}\alpha_{IS}}{\beta_{IS} + \beta_{LM}}$ (21) **Equilibrium interest rate.**

- **<u>Including inflation in the IS-LM model.</u>**

Recall that $i = r + \pi^e$, which implies $r = i - \pi^e$. Substitute the latter expression into (11) and we have modified IS-LM equations, expressed in terms of the **<u>nominal interest rate (i)</u>**.

$\mathbf{i} = (\pi^e + \alpha_{IS}) - \beta_{IS}\mathbf{Y}$ (22) Modified IS equation.

$\mathbf{i} = \alpha_{LM} + \beta_{LM}\mathbf{Y}$ (16) LM equation from (16).

This modification allows us to analyze the economic effects of changes in expected inflation.

Source: Macroeconomics, fourth edition (2001) by Abel & Bernanke.