Problem Number 1

A farmer can plant up to 8 acres of land with wheat and barley. He can earn $5,000 for every acre he plants with wheat and $3,000 for every acre he plants with barley. His use of a necessary pesticide is limited by federal regulations to 10 gallons for his entire 8 acres. Wheat requires 2 gallons of pesticide for every acre planted and barley requires just 1 gallon per acre.

What is the maximum profit he can make?

Solution to Problem Number 1

let x = the number of acres of wheat
let y = the number of acres of barley.

since the farmer earns $5,000 for each acre of wheat and $3,000 for each acre of barley, then the total profit the farmer can earn is 5000*x + 3000*y.

let p = total profit that can be earned, your equation for profit becomes:

p = 5000x + 3000y
that's your objective function, it's what you want to maximize.

the constraints are:
number of acres has to be greater than or equal to 0.
number of acres has to be less than or equal to 8.
amount of pesticide has to be less than or equal to 10.

your constraint equations are:
x >= 0
y >= 0
x + y <= 8
2x + y <= 10

to graph these equations, solve for y in those equations that have y in them and then graph the equality portion of those equations.

x >= 0
y >= 0
y <= 8-x
y <= 10 - 2x

x = 0 is a vertical line that is the same line as the y-axis.
y = 0 is a horizontal line that is the same line as the x-axis.

the area of the graph that satisfies all the constraints is the region of feasibility.

the maximum or minimum solutions to the problem will be at the intersection points of the lines that bound the region of feasibility.

the graph of your equations looks like this:
the region of feasibility is the shaded area of the graph.

you can see from this graph that the region of feasibility is bounded by the following \((x,y)\) coordinate points:

\((0,0)\)
\((0,8)\)
\((2,6)\)
\((5,0)\)

the point \((0,0)\) is the intersection of the line \(x\)-axis with the \(y\)-axis.
the point \((0,8)\) is the intersection of the line \(y = 8 - x\) with the \(y\)-axis.
the point \((5,0)\) is the intersection of the line \(y = 10 - 2x\) with the \(x\)-axis.
the point \((2,6)\) is the intersection of the line \(y = 8 - x\) with the line \(y = 10 - 2x\).

the point \((2,6)\) was solved for in the following manner:
equations of the intersecting lines are:
\(y = 8 - x\)
\(y = 10 - 2x\)
subtract the first equation from the second equation and you get:
\(0 = 2 - x\)
add \(x\) to both sides of this equation and you get:
\(x = 2\)
substitute 2 for \(x\) in either equation to get \(y = 6\).
that makes the intersection point \((x,y) = (2,6)\).

the objective equation is:
\[ p = 5000x + 3000y \]

profit will be maximum at the intersection points of the region of feasibility on the graph.
the profit equation is evaluated at each of these points as shown in the following table.

<table>
<thead>
<tr>
<th>intersection point of ((x,y))</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0,0))</td>
<td>$0</td>
</tr>
<tr>
<td>((0,8))</td>
<td>$24,000</td>
</tr>
<tr>
<td>((2,5))</td>
<td>$28,000</td>
</tr>
<tr>
<td>((5,0))</td>
<td>$25,000</td>
</tr>
</tbody>
</table>

the maximum profit occurs when the farmer plants 2 acres of wheat and 6 acres of barley.
number of acres of wheat is 2 and number of acres of barley is 6 for a total of 8 acres which is the maximum number of acres available for planting.
number of gallons of pesticide used for wheat is 4 and number of gallons of pesticide used for barley is 6 for a total of 10 gallons of pesticide which is the maximum amount of pesticide that can be used.

PROBLEM NUMBER 2

A painter has exactly 32 units of yellow dye and 54 units of green dye.
He plans to mix as many gallons as possible of color A and color B.
Each gallon of color A requires 4 units of yellow dye and 1 unit of green dye.
Each gallon of color B requires 1 unit of yellow dye and 6 units of green dye.

Find the maximum number of gallons he can mix.

SOLUTION TO PROBLEM NUMBER 2

the objective function is to determine the maximum number of gallons he can mix.

the colors involved are color A and color B.

let \(x\) = the number of gallons of color A.
let \(y\) = the number of gallons of color B.

if we let \(g\) = the maximum gallons the painter can make, then the objective function becomes:

\[ g = x + y \]

make a table for color A and color B to determine the amount of each dye required.
your table will look like this:

each gallon of color A or B will require:

<table>
<thead>
<tr>
<th></th>
<th>units of yellow dye</th>
<th>units of green dye</th>
</tr>
</thead>
<tbody>
<tr>
<td>color A</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>color B</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

total units of yellow dye available are 32
total units of green dye available are 54

your constraint equations are:
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ 4x + y \leq 32 \]
\[ x + 6y \leq 54 \]

\( x \) and \( y \) have to each be greater than or equal to 0 because the number of gallons can't be negative.

in order to graph these equations, you solve for \( y \) in those equations that have \( y \) in them.

the equations for graphing are:
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ y \leq 32 - 4x \]
\[ y \leq (54 - x)/6 \]

\( x = 0 \) is a vertical line that is the same line as the \( y \)-axis.
\( y = 0 \) is a horizontal line that is the same line as the \( x \)-axis.

the graph will look like this:
the region of feasibility is the shaded area of the graph.
all points within the feasibility region meet the constraint of the problem.

the intersection points of the region of feasibility are:
(0,0)
(0,9)
(6,8)
(8,0)

the maximum or minimum value of the objective function will be at these points of intersection.

solve the objective function at each of these intersection points to determine which point contains the maximum number of gallons.
the objective function is:

\[ g = x + y \]

the table with the value of \( g \) at each of these intersection points is shown below:

<table>
<thead>
<tr>
<th>intersection point ((x, y))</th>
<th>gallons of paint</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>0</td>
</tr>
<tr>
<td>((0, 9))</td>
<td>9</td>
</tr>
<tr>
<td>((6, 8))</td>
<td>14</td>
</tr>
<tr>
<td>((8, 0))</td>
<td>8</td>
</tr>
</tbody>
</table>

the maximum gallons of paint for color A and B, given the constraints, is equal to 14.

this is comprised of 6 gallons of color A and 8 gallons of color B.

6 gallons of color A uses 24 gallons of yellow dye and 8 gallons of color B uses 8 gallons of yellow dye for a total of 32 gallons of yellow dye which is the maximum amount of yellow dye that can be used.

6 gallons of color A uses 6 gallons of green dye and 8 gallons of color B uses 48 gallons of green dye for a total of 54 gallons of green dye which is the maximum amount of green dye that can be used.

**PROBLEM NUMBER 3**

The Bead Store sells material for customers to make their own jewelry. Customer can select beads from various bins. Grace wants to design her own Halloween necklace from orange and black beads. She wants to make a necklace that is at least 12 inches long, but no more than 24 inches long. Grace also wants her necklace to contain black beads that are at least twice the length of orange beads. Finally, she wants her necklace to have at least 5 inches of black beads.

Find the constraints, sketch the problem and find the vertices (intersection points)

**SOLUTION TO PROBLEM NUMBER 3**

let \( x \) = the number of inches of black beads,

let \( y \) = the number of inches of orange beads.

the objective function is the length of the necklace

there is a maximum length and a minimum length.

if you let \( n \) equal the length of the necklace, then the objective function becomes:

\[ n = x + y \]

since the problem is looking for the number of inches of black beads and the number of inches of orange beads, we will let:
the constraint equations for this problem are:
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ x + y \geq 12 \]
\[ x + y \leq 24 \]
\[ x \geq 2y \]
\[ x \geq 5 \]

\[ x \geq 0 \] is there because the number of inches of black beads can't be negative.
\[ y \geq 0 \] is there because the number of inches of orange beads can't be negative.
\[ x + y \geq 12 \] is there because the total length of the necklace has to be greater than or equal to 12 inches.
\[ x + y \leq 24 \] is there because the total length of the necklace has to be less than or equal to 24 inches.
\[ x \geq 2y \] is there because the length of the black beads has to be greater than or equal to twice the length of the orange beads.
\[ x \geq 5 \] is there because the number of inches of black beads has to be greater than or equal to 5.

to graph these equations, we have to solve for \( y \) in each equation that has \( y \) in it and then graph the equality portion of each of them.

your equations for graphing are:
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ y \geq 12 - x \]
\[ y \leq 24 - x \]
\[ y \leq x/2 \]
\[ x \geq 5 \]

\[ x = 0 \] is a vertical line that is the same line as the \( y \)-axis.
\[ y = 0 \] is a horizontal line that is the same line as the \( x \)-axis.
\[ x = 5 \] is a vertical line at \( x = 5 \).

a graph of you equations is shown below:
the region of feasibility is the shaded area of the graph.
all points within the region of feasibility meet the constraint requirements of the problem.

the intersection points bounding the region of feasibility are:
(8,4)
(12,0)
(16,8)
(24,0)

(8,4) is the intersection of the lines $y = x/2$ and $y = 12 - x$
to find the point of intersection, set $x/2$ and $12-x$ equal to each other and solve for $x$.
you get:
$x/2 = 12-x$
multiply both sides of the equation by 2 to get:
\[ x = 24 - 2x \]
add 2x to both sides of the equation to get:
\[ 3x = 24 \]
divide both sides of the equation by 3 to get:
\[ x = 8 \]
substitute 8 for x in either equation to get \[ y = 4 \].

(12,0) is the intersection of the line \[ y = 12 - x \] with the x-axis.
(24,0) is the intersection of the line \[ y = 24 - x \] with the x-axis.
to find the point of intersection, set \[ y \] equal to 0 in each equation and solve for \[ x \].

(16,8) is the intersection of the lines \[ y = x/2 \] and \[ y = 24 - x \].
to find the intersection point, set \[ x/2 \] equal to 24-x and solve for \[ x \].
you get:
\[ x/2 = 24 - x \]
multiply both sides of this equation by 2 to get:
\[ x = 48 - 2x \]
add 2x to both sides of this equation to get:
\[ 3x = 48 \]
divide both sides of this equation by 3 to get:
\[ x = 16 \]
substitute 16 for \[ x \] in either equation to get:
\[ y = 8 \].

the maximum / minimum necklace length will be at the intersection points of the boundaries of the region of feasibility.

evaluation of the objective function at these intersections yields the following:
objective function is:
\[ x + y = n \] where \( n \) is the length of the necklace in inches.

<table>
<thead>
<tr>
<th>Intersection points</th>
<th>Number of inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9, 4)</td>
<td>12</td>
</tr>
<tr>
<td>(12, 0)</td>
<td>12</td>
</tr>
<tr>
<td>(16, 8)</td>
<td>24</td>
</tr>
<tr>
<td>(24, 0)</td>
<td>24</td>
</tr>
</tbody>
</table>

the number of inches of black beads is at least twice the number of inches of orange beads.
the number of inches of black beads is at least 5.
the total length of the necklace is greater than or equal to 12 inches or less than or equal to 24 inches.
all the constraints have been met.
the maximum length the necklace can be and still meet the constraints is 24 inches.
the minimum length the necklace can be and still meet the constraints is 12 inches.

PROBLEM NUMBER 4

A garden shop wishes to prepare a supply of special fertilizer at a minimal cost by mixing two fertilizers, A and B.
The mixture is to contain:
at least 45 units of phosphate
at least 36 units of nitrate
at least 40 units of ammonium
Fertilizer A costs the shop $.97 per pound.
Fertilizer B costs the shop $1.89 per pound.
fertilizer A contains 5 units of phosphate and 2 units of nitrate and 2 units of ammonium.
fertilizer B contains 3 units of phosphate and 3 units of nitrate and 5 units of ammonium.

how many pounds of each fertilizer should the shop use in order to minimize their cost.

SOLUTION TO PROBLEM NUMBER 4

let \( x \) = the number of pounds of fertilizer A.
let \( y \) = the number of pounds of fertilizer B.

the objective function is to minimize the cost.

the objective function becomes:

\[ c = .97x + 1.89y \]

the constraint equations are:

since the number of pounds of each fertilizer can't be negative, 2 of the constraint equations become:

\[ x \geq 0 \]
\[ y \geq 0 \]

since the number of units of phosphate has to be at least 45, the constraint equation for phosphate becomes:

\[ 5x + 3y \geq 45 \]

since the number of units of nitrate must be at least 36, the constraint equation for nitrates becom
2x + 3y >= 36

since the number of units of ammonium must be at least 40, the constraint equation for ammonium becomes:

2x + 5y >= 40

the constraint equations for this problem become:

x >= 0
y >= 0
5x + 3y >= 45
2x + 3y >= 36
2x + 5y >= 40

in order to graph these equations, you have to solve for y in each equation that has y in it and then graph the equality portion of those equations.

the equations to be graphed are:

x >= 0
y >= 0
y >= (45-5x)/3
y >= (36 - 2x)/3
y >= (40-2x)/5

x = 0 is a vertical line that is the same line as the y-axis.
y = 0 is a horizontal line that is the same line as the x-axis.

a graph of your equation is shown below:
the feasibility region is the area of the graph that is shaded. all points within the region of feasibility meet the constraint requirements of the problem.

the intersection points at the boundaries of the feasibility region are:

(0, 15)
(3, 10)
(15, 2)
(20, 0)

the intersection points of the boundaries of the region of feasibility contain the minimum cost solution for the objective function in this problem.

now that you have the intersection points, you can solve for the minimum cost equation which is the o
Objective function of:

\[ c = 0.97x + 1.89y \]

The following table shows the value of the cost equation at each of the intersection points.

<table>
<thead>
<tr>
<th>intersection points (x,y)</th>
<th>c = 0.97x + 1.89y</th>
<th>minimum solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,15)</td>
<td>28.35</td>
<td></td>
</tr>
<tr>
<td>(3,10)</td>
<td>21.81</td>
<td></td>
</tr>
<tr>
<td>(15,2)</td>
<td>18.33</td>
<td>******</td>
</tr>
<tr>
<td>(20,0)</td>
<td>18.40</td>
<td></td>
</tr>
</tbody>
</table>

The table suggests that we have a minimum cost solution when the value of \( x \) is equal to 15 and the value of \( y \) is equal 2.

When \( x = 15 \) and \( y = 2 \), the number of pounds of potassium, nitrates, and ammonium are:

- Phosphate: \( 5x + 3y = 5*15 + 3*2 = 75 + 6 = 81 \)
- Nitrate: \( 2x + 3y = 2*15 + 3*2 = 30 + 6 = 36 \)
- Ammonium: \( 2x + 5y = 2*15 + 5*2 = 30 + 10 = 40 \)

All the constraints associated with the minimum cost objective have been met.

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Linear Programming I: Maximization
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Assignment 10 is on the last page.

Learning objectives:

1. Recognize problems that linear programming can handle.
2. Know the elements of a linear programming problem -- what you need to calculate a solution.
3. Understand the principles that the computer uses to solve a linear programming problem.
4. Understand, based on those principles:
   a. Why some problems have no feasible solution.
   b. Why non-linearity requires much fancier technique.
5. Be able to solve small linear programming problems yourself.

Linear programming intro

Linear programming is constrained optimization, where the constraints and the objective function are all linear. It is called "programming" because the goal of the calculations help you choose a "program" of action.

Classic applications:

1. Manufacturing -- product choice
   Several alternative outputs with different input requirements
   Scarcity inputs
   Maximize profit

2. Agriculture -- feed choice
   Several possible feed ingredients with different nutritional content
   Nutritional requirements
   Minimize costs

3. "The Transportation Problem"
   Several depots with various amounts of inventory
   Several customers to whom shipments must be made
   Minimize cost of serving customers

4. Scheduling
   Many possible personnel shifts
   Staffing requirements at various times
   Restrictions on shift timing and length
   Minimize cost of meeting staffing requirements
5. Finance
   Several types of financial instruments available
   Cash flow requirements over time
   Minimize cost

Start with
The Manufacturing Problem -- Example 1

A manufacturer makes wooden desks (X) and tables (Y). Each desk requires 2.5 hours to assemble, 3 hours for buffing, and 1 hour to crate. Each table requires 1 hour to assemble, 3 hours to buff, and 2 hours to crate. The firm can do only up to 20 hours of assembling, 30 hours of buffing, and 16 hours of crating per week. Profit is $3 per desk and $4 per table. Maximize the profit.

The linear programming model, for a manufacturing problem, involves:

Processes or activities that can be done in different amounts

Constraints -- resource limits

The constraints describe the production process -- how much output you get for any given amounts of the inputs.

The constraints say that you cannot use more of each resource than you have of that resource.

Linear constraints means no diminishing or increasing returns. Adding more input gives the same effect on output regardless of how much you are already making.

Non-negativity constraints -- the process levels cannot be less than 0. This means that you cannot turn your products back into resources.

Objective function -- to be maximized or minimized

A linear objective function means that you can sell all you want of your outputs without affecting the price. You have elastic demand, in economics jargon.

Translating the words of Example 1 into equations (which is not a trivial task), we have this:

The objective function is Profit = 3x + 4y
x is the number of desks  y is the number of tables

Constraints:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>assembling</td>
<td>2.5x + y ≤ 20</td>
<td>The total assembling time, for example, is the time spent assembling desks plus the time spent assembling tables. That's 2.5x + 1y. This must be less or equal to the 20 hours available.</td>
</tr>
<tr>
<td>buffing</td>
<td>3x + 2y ≤ 30</td>
<td></td>
</tr>
<tr>
<td>crating</td>
<td>x + 2y ≤ 16</td>
<td></td>
</tr>
<tr>
<td>non-negativity</td>
<td>x ≥ 0, y ≥ 0</td>
<td></td>
</tr>
</tbody>
</table>
Graphical method of solution – for maximization

One way to solve a linear programming problem is to use a graph. The graph method lets you see what is going on, but its accuracy depends on how careful a draftsman you are.

1. Plot the constraints. Express each constraint as an equation. (Change the ≤ or ≥ to an =.)

2. Find the feasible region. It's below all lines for constraints that are ≤ and above all lines for constraints that are ≥.

Here, the feasible region is below the 2.5x+y=20, 3x+3y=30, and x+2y=16 lines – those are the ≤ constraints. It's above the y=0 line (the x-axis), and to the right of the x=0 line (the y-axis) – those are the ≥ constraints.

3. Superimpose isoprofit lines.

To get an isoprofit line, set the objective function equal to some arbitrary number. That gives you a linear equation in X and Y that you can plot on your graph. "Iso" means "the same." All the points on an isoprofit line give the same profit.

As a first try, in the diagram to the right, I drew an isoprofit line for a profit of 45. The equation for the line is 3x+4y=45. That is because we make $3 for each desk and $4 for each table. 3x+4y=45 is a line containing all the points that have a profit of exactly 45.

That 45 isoprofit line happened to be higher than the feasible region in the lower left corner, bounded by the constraints.

4. Find the highest value isoprofit line that touches the feasible region. Imagine moving that 3x+4y=45 line, parallel to itself, down and to the left. Move it down a little bit and you might have the line 3x+4y=44. Move it some more and you might have the line 3x+4y=43. Keep going until your line just touches a corner of the feasible area. Stop there, and you have the line 3x+4y=36, which is shown in the diagram.
LINEAR PROGRAMMING I

All isoprofit lines have the same slope. They differ only in how high they are. The slope of an isoprofit line depends on the ratio of the x good's profit-per-unit to y good's profit-per-unit.

The profit amount for the isoprofit line that just touches the feasible area is the most profit you can make. The X and Y coordinates of the point where the isoprofit line touches tells you how much of x and y to make.

Why do you stop with the the isoprofit line that just touches the feasible area? Higher isoprofit lines have no feasible points on them. We cannot use those. Lower isoprofit lines, that touch more of the feasible area, have less profit than the one that just touches a corner. We want the most profit that is feasible, so we want the line that just touches a corner.

The solution is $x = 4$, $y = 6$, and the profit is 36.

Enumeration method of solution

The enumeration method is another way of solving a linear programming problem. It gives an exact answer that does not depend on your drawing ability. However, it can involve a lot of calculating. To understand the enumeration method, we start with the graph method.

The feasible region in the diagram above is convex with straight edges. This is always true in linear programming problems. This implies the extreme point theorem: If a feasible region exists, the optimal point will be a corner of the feasible region. When the isoprofit line just touches the feasible region, it will be touching at a corner. (It is possible for the isoprofit line to touch a whole edge, if one of the constraint lines is parallel to the isoprofit lines. That whole edge will include two corners, so the general theorem about corners still applies.)

The corners of the feasible region are points where constraints intersect. If we can find all of the intersections of the constraints, we know that one of those intersections must be an optimal point.

Some jargon: Each point where the constraints intersect is called a basic solution.

A basic solution for a 2-product problem like ours is where any two of the constraint lines intersect.

(If our problem had three products, each basic solution would be where three of the constraint planes intersect. For a problem with $n$ products, where $n$ is more than 3, each basic solution would be where $n$ of the constraint hyperplanes intersect.)

The non-negativity equations count as constraints, too, when identifying basic solutions. Example 1 has 5 constraints, and therefore 10 basic solution intersections. They are shown in the diagram.

Some of the basic solutions are corners of the
LINEAR PROGRAMMING I

feasible region. Other basic solutions are outside of the feasible area.

A basic feasible solution is a basic solution that satisfies all the constraints. In other words, a basic feasible solution is a corner of the feasible area. In Example 1 there are 5 basic feasible solutions, the five corners of the feasible region.

The extreme point theorem implies that one of the basic feasible solutions is the optimal point.

To get the solution to this linear programming problem, we just have to test all of the basic solutions to see which are feasible and which are not. Once we have the basic feasible solution, we calculate the profit for each one and pick the highest.

Summarizing the enumeration method (for $n$ dimensions, representing $n$ different products):

1. Make all possible groups of $n$ equations from the constraints.
2. Solve each group of equations. This gives you all the basic solutions.
3. Test each solution against the other constraints.
4. Throw out any solutions that aren’t feasible. The ones that are left are the basic feasible solutions.
5. Calculate the profit for each basic feasible solution.
6. Pick the highest profit point as your answer.

This can be done entirely with algebra. You don’t need to draw a diagram.

Enumeration solution to Example 1:

$n=2$, so we solve the equations in pairs. There are 5 constraint equations, counting the non-negativity conditions, so there are 10 possible pairs of equations. That means 10 solutions.

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Slack in the Feasible Constraints</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
<td>$20$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$12$</td>
</tr>
<tr>
<td>$0$</td>
<td>$10$</td>
<td>$10$</td>
</tr>
<tr>
<td>$0$</td>
<td>$20$</td>
<td>$0$</td>
</tr>
<tr>
<td>$4$</td>
<td>$6$</td>
<td>$4$</td>
</tr>
<tr>
<td>$6$</td>
<td>$5$</td>
<td>$0$</td>
</tr>
<tr>
<td>$6.667$</td>
<td>$3.333$</td>
<td>$0$</td>
</tr>
<tr>
<td>$8$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$10$</td>
<td>$0$</td>
<td>$-5$</td>
</tr>
<tr>
<td>$16$</td>
<td>$0$</td>
<td>$-20$</td>
</tr>
</tbody>
</table>

The table above lists all ten intersections, the ten basic solutions. The middle columns test to see if each one uses too many resources. Negative numbers under “Slack in the Constraints” indicate that the solution uses too much of a resource, and is therefore not feasible. For the solutions that are feasible, the profit is calculated. The winner is the row with the highest profit.
The advantages of the enumeration method over the graphics method are:
1. Enumeration can be used for problems with more than 2 dimensions.
2. It gives an exact answer.
3. You can program a computer to do it, because it is entirely algebraic.

The disadvantage of the enumeration method:
The math grows rapidly with number of equations. If you have \( c \) constraints and \( p \) processes, the number of intersections is \( (c+p)! / c!p! \). For example, if you have three constraints and two variables, you have ten intersections. If you have six constraints and six variables, you have 924 intersections. And each intersection requires that much more calculation to find.

**Simplex method -- for maximization**

The simplex method is so named because the shape of the feasible region, a solid bounded by flat planes, is called a "simplex."

The simplex method is an algorithm. It speeds up the enumeration method by moving step-by-step from one basic feasible solution to another with higher profit until the best is found. The simplex method is presented here for your edification. You will not be asked to do the simplex method by hand like this. The idea of this presentation is to show you that the simplex method has a series of steps that are suitable for a computer to do.

The simplex method requires first putting the problem into a standard form. You change the constraints from inequalities into equalities by adding slack (or surplus) variables. You add one slack variable for each constraint. It represents how much of your capacity in that particular regard that you aren't using.

Example 1 constraints become

\[
2.5x + y + s_1 = 20 \\
3x + 3y + s_2 = 30 \\
x + 2y + s_3 = 16
\]

We want to express these equations in matrix form. To do this, we include all the variables in every equation, putting in 0's where needed, like this:

\[
egin{align*}
2.5x + 1y + 1s_1 + 0s_2 + 0s_3 &= 20 \\
3x + 3y + 0s_1 + 1s_2 + 0s_3 &= 30 \\
x + 2y + 0s_1 + 0s_2 + 1s_3 &= 16
\end{align*}
\]

This system of three equations in five unknowns has 10 solutions. Each one corresponds to one of the basic feasible solutions shown on pages 4 and 5. One of those 10 is relatively easy to spot. Suppose, in the above equations, \( x \) and \( y \) are both 0. Then the three equations reduce to \( 1s_1 = 20 \), \( 1s_2 = 20 \), and \( 1s_3 = 16 \). This is the solution corresponding to making no desks or tables and leaving all of the resources unused ("slack").
1. A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only $1200 to spend and each acre of wheat costs $200 to plant and each acre of rye costs $100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is $500 per acre of wheat and $300 per acre of rye how many acres of each should be planted to maximize profits?

2. A gold processor has two sources of gold ore, source A and source B. In order to keep his plant running, at least three tons of ore must be processed each day. Ore from source A costs $20 per ton to process, and ore from source B costs $10 per ton to process. Costs must be kept to less than $80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?

3. A publisher has orders for 600 copies of a certain text from San Francisco and 400 copies from Sacramento. The company has 700 copies in a warehouse in Novato and 800 copies in a warehouse in Lodi. It costs $5 to ship a text from Novato to San Francisco, but it costs $10 to ship it to Sacramento. It costs $15 to ship a text from Lodi to San Francisco, but it costs $4 to ship it from Lodi to Sacramento. How many copies should the company ship from each warehouse to San Francisco and Sacramento to fill the order at the least cost?