Seven Proofs to Know: at least one of these will be on Exam 4.

Each student is assigned a proof to present in class during the last week of classes.

1. (Edwin) Let f be a function from A to B, and let g be a function from B to C.
   Prove: If f and g are both 1-to-1 functions, then $g \circ f$ is a 1-to-1 function from A to C.

2. (Nathan) Let $f: N \to Z$ by the rule $f(n) = \left\{ \begin{matrix} -n/2, \text{ if n is even} \\ (n+1)/2, \text{ if n is odd} \end{matrix} \right\}$.
   Find the image of f and prove, in full detail, that your answer is correct.

3. (Thomas) Let a and b be real numbers with $a > 1$ and $b > 1$. First, explain why $\log_a(b) > 0$. Then, use that fact to prove: $\log_a(n) \in \Theta(\log_b(n))$

4. (Adrian) Suppose A and B are events in a sample space (S, P).
   a. Disprove: If $P(A) \le P(B)$, then $A \subseteq B$;
   b. Use specific definitions and/or probability theorems to prove:
      If $A \subseteq B$, then $P(A) \le P(B)$.

5. (Bre) Let A and B be events in a sample space. Prove: If A and B are independent, then A and $\overline{B}$ are independent.

6. (Matt) Let X be a random variable on sample space (S, P). Prove: If X is not a constant function, then X is not independent of itself.

7. (John) Let a and b be positive integers. Prove, using basic definitions: b divides a iff $a \ div \ b = \frac{a}{b}$.