To give students exposure to more than one example of hypothesis testing, it's often worth the effort to use a different example in class than we used in the text. For instance, consider a supply equation for cotton:

\[ C_t = f(AH_t, PC_{t-1}, PS_{t-1}, M_t) + \varepsilon_t \]

\[ C_t = \beta_0 + \beta_1 AH_t + \beta_2 PC_{t-1} + \beta_3 PS_{t-1} + \beta_4 M_t + \varepsilon_t \]

where:
- \( C_t \) = thousands of bales of cotton produced in year \( t \)
- \( AH_t \) = thousands of acres of cotton harvested in year \( t \)
- \( PC_{t-1} \) = the real price of a thousand pounds of cotton in year \( t - 1 \)
- \( PS_{t-1} \) = the real price of a thousand bushels of soybeans in year \( t - 1 \)
- \( M_t \) = price index for farm machinery in year \( t \)

This is a typical student effort at such an equation. Acres harvested is not tautologically related to production (as acres planted might be if technology was constant during the sample), but it is likely to be jointly determined with production. Still, a positive sign would be expected. The price of cotton is lagged one time period because of the length of time it takes to switch crops; a positive sign is expected. Soybeans can be planted on the same kinds of land on which cotton is planted, and so a negative sign is expected. Finally, the price of cotton machinery (negative expected sign) was unavailable, so the student instead chose the average price of all farm machinery, leading to an ambiguous expected sign (since cotton is fairly labor intensive, an increase in price in farm machinery might shift farmers into cotton production).

Based on this theory, the appropriate null and alternative hypotheses would be:

- \( H_0: \beta_i \leq 0 \) (null hypothesis)
- \( H_A: \beta_i > 0 \) (alternative hypothesis)

5-2. The \( t \)-Test

Continuing on with the cotton example, the appropriate border value for all four null hypotheses is zero, so the formula for the \( t \)-statistic to use is Equation 5.3:

\[ t_k = \frac{\hat{\beta}_k}{SE(\hat{\beta}_k)} \]

The equation will be estimated for annual U.S. data from 1947 through 1972, so there will be 21 degrees of freedom. Given this, and a 5% level of significance, the appropriate decision rules and critical \( t \)-values (from Statistical Table B.1) are:  one-sided test: \( 5\% \), \( t^* = 1.721 \)

5-3. Examples of \( t \)-Tests

Continuing on with the cotton example, the estimated equation is:

\[ \hat{C}_t = 624 + 0.33 AH_t + 1929 PC_{t-1} - 154 PS_{t-1} + 12.4 M_t \]

\[ \text{STD. ERR} (0.10) (1078) (82) (7.4) \]

\[ t = 3.16 \quad 1.79 \quad -1.89 \quad 1.67 \]

(These \( t \)-scores are computer-calculated and differ from those calculated by hand because of rounding.)
Computed t-stat for $\hat{B}_i = 2.37$ \( \frac{\hat{B}_i}{\text{var. Error}} = \frac{1.28}{0.54} \)

This means $\hat{B}_i$ is 2.37 standard errors away from zero.

Confidence Intervals

- d.f. $33 - 1 - 3 = 29$

- Critical t-values:
  - Two-sided:
    - $t^*_{5\%} = 2.045$
    - $t^*_{1\%} = 2.78$