Second Exam Answers
Phi 321 Formal Logic
Fall 2015, Jared Bates

Part 1. Translations. Translate the following sentences into PLE using the symbolization guides provided. (10 pts each)

1. Ajax, a son of Telemon, is an Achaean hero. (Sxy: x is a son of y. Ax: x is an Achaean. Hx: x is a hero. a: Ajax t: Telemon.)

   Sat & (Aa & Ha)

2. No Achaean fears Paris. (Ax: x is an Achaean. Fxy: x fears y. p: Paris)

   (∀x)(Ax ⊃ ~Fxp) --or-- ~(∃x)(Ax & Fxp)

3. Everyone except Achilles is fighting some Trojan. (Fxy: x is fighting y. Tx: x is a Trojan. a: Achilles)

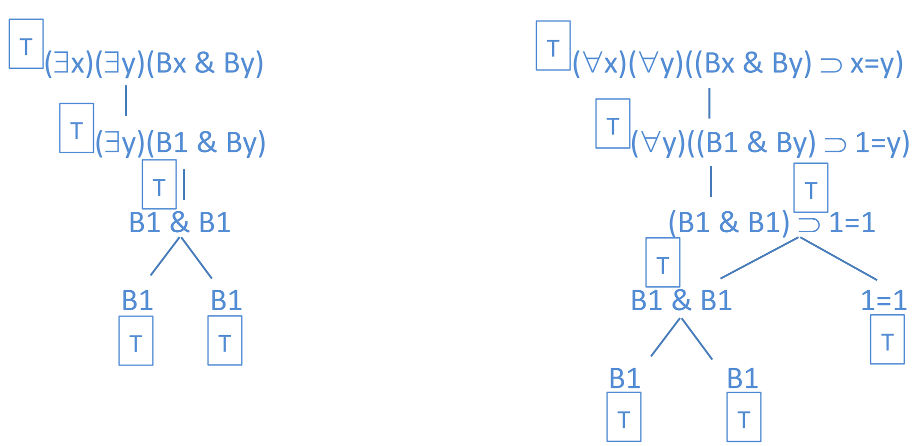
   (∀x)(x≠a ⊃ (∃y)(Ty & Fxy))

4. The Trojan with a flashing helmet is a brother of Paris. (Tx: x is a Trojan. Fx: x has a flashing helmet. Bxy: x is a brother of y. p: Paris)

   (∃x)((∀y)((Ty & Fy) ≡ y=x) & Bxp)

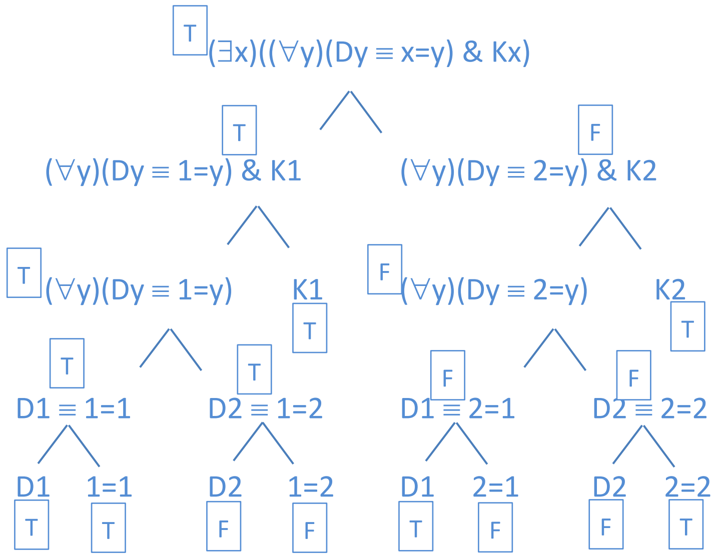
Part 2. Interpretations. Construct interpretations to solve the following problems. Use interpretations with small, numerical domains, and verify your interpretations with full semantic trees. (10 pts each)

5. Show that the following set of sentences is quantificationally consistent.
   (∃x)(∃y)(Bx & By) (∀x)(∀y)((Bx & By) ⊃ x=y)

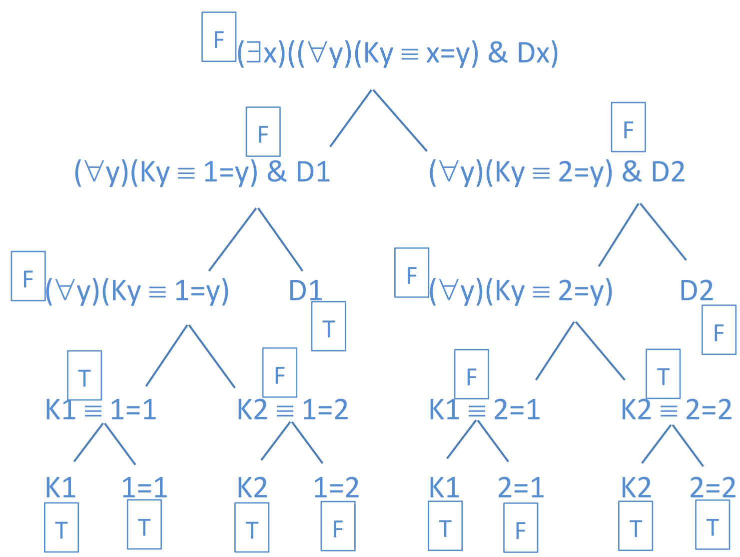


UD: {1}
B: {1}

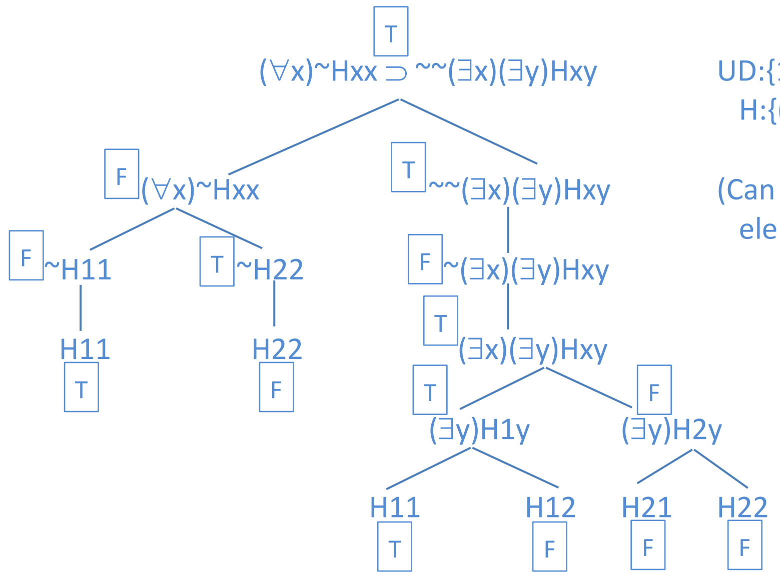
6. Show that the following pair of sentences is not quantificationally equivalent.
   (∃x)((∀y)(Dy ≡ x=y) & Kx) (∃x)((∀y)(Ky ≡ x=y) & Dx)



UD: {1,2}
K: {1,2}
D: {1}



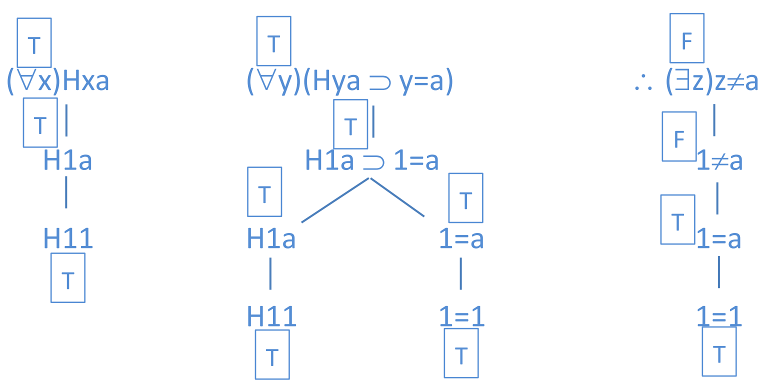
7. Show that the following sentence is not quantificationally false.
   (∀x)~Hxx ⊃ ~~(∃x)(∃y)Hxy



UD:{1,2}
H:{(1,1)}

(Can be done with a one-element domain.)

8. Show that the following argument is quantificationally invalid.
   (∀x)Hxa. (∀y)(Hya ⊃ y=a). ∴ (∃z)z≠a



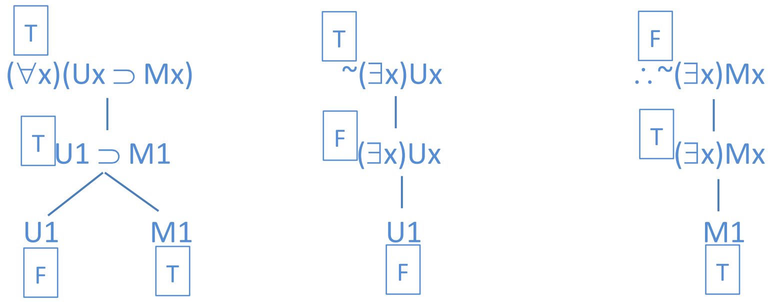
UD: {1}
H: {(1,1)}

Part 3. Combination. Translate the following argument into PLE using the symbolization guide provided. Then prove that your translation is *quantificationally invalid*. Provide an interpretation with a small, numerical domain, and verify your interpretation with full semantic trees. (20 pts)

9. All unicorns have monkey paws. There are no unicorns. So, nothing has monkey paws.
   (Ux: x is a unicorn. Mx: x has monkey paws.)

TRANSLATION: $(\forall x)(Ux \supset Mx)$. $\sim(\exists x)Ux$. $\therefore \sim(\exists x)Mx$ QUANTIFICATIONALLY INVALID.

PROOF:



UD: {1}
U: {}
M: {1}