First Exam Answers

Phi 321 Formal Logic

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Part 1. Translations. Symbolize the following sentences in SL using the scheme of abbreviations provided. (10 pts each)

1. Hector and Paris will not both survive the war. (H: Hector will survive the war. P: Paris will survive the war.)

   ~(H & P) --or-- ~H v ~P

2. Achilles will not join the fight unless either Agamemnon is punished or Patroclus is slain. (F: Achilles will join the fight. P: Agamemnon is punished. S: Patroclus is slain.)

   ~F v (P v S) --or-- ~(P v S) ⊃ ~F --or-- F ⊃ (P v S)

Part 2. Truth Tables. Construct <u>full</u> truth tables to answer the following questions. (10 pts each)

3. Is the following sentence truth-functionally true, false, or indeterminate?
   ((B ⊃ S) & (B ⊃ ~S)) ⊃ ~B

						v	
B	S	((B ⊃ S)	&	(B ⊃	~S)	⊃	~B
T	T	T	F	F	F	T	F
T	F	F	F	T	T	T	F
F	T	T	T	T	F	T	T
F	F	T	T	T	T	T	T

Truth-functionally true

4. Is the following set of sentences truth-functionally consistent?
   R ⊃ (N v H). (N & R) & ~H

			v			v	
H	N	R	R ⊃	(N v H)	(N & R)	&	~H
F	T	T	T	T	T	T	T

I am not presenting the 8-row truth-table here. The set is <u>truth-functionally consistent</u>. Here is the only truth-value assignment on which all members of the set are true.

5. Is the following argument truth-functionally valid? If not, indicate a counterexample.
   ~(K v M). ~C. ∴ K ≡ C

			v		v	v
C	K	M	~(K	v M)	~C	∴ K ≡ M
F	F	F	T	F	T	T

Again, I'm only presenting a shortened table here. The argument is <u>truth-functionally valid</u>. There is only one truth-value assignment on which both premises are true. The conclusion is true there as well, so there is no counterexample.

Part 3. Derivations. Demonstrate the validity of the following arguments by constructing derivations. (10 pts each)

6. S ≡ (~I v D). G ⊃ (~A & D). ~M & G. ∴ (S & ~M) v ~~J

DERIVE: (S & ~M) v ~~J

This proof uses only SD rules.

```
1.  | S ≡ (~I v D)          A
2.  | G ⊃ (~A & D)          A
3.  | ~M & G                A
    |---------------------
4.  | G                     3 &E
5.  | ~A & D                2,4 ⊃E
6.  | D                     5 &E
7.  | ~I v D                6 vI
8.  | S                     1,7 ≡E
9.  | ~M                    3 &E
10. | S & ~M                8,9 &I
11. | (S & ~M) v ~~J        10 vI
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7. ∴ ~(D & S) ≡ (S ⊃ ~D)

DERIVE: ~(D & S) ≡ (S ⊃ ~D)

This proof uses only SD rules.

```
1.  | | ~(D & S)                A/≡I
    | |---------------------
2.  | | | S                     A/⊃I
    | | |-------------------
3.  | | | | D                   A/~I
    | | | |-----------------
4.  | | | | D & S               2,3 &I
5.  | | | | ~(D & S)            1 R
6.  | | | ~D                    3-5 ~I
7.  | | S ⊃ ~D                  2-6 ⊃I
8.  | | S ⊃ ~D                  A/≡I
    | |---------------------
9.  | | | D & S                 A/~I
    | | |-------------------
10. | | | S                     9 &E
11. | | | ~D                    8,10 ⊃E
12. | | | D                     9 &E
13. | | ~(D & S)                9-12 ~I
14. | ~(D & S) ≡ (S ⊃ ~D)       1-7,8-13 ≡I
```

DERIVE: ~(D & S) ≡ (S ⊃ ~D)

And now for some fancy SD+ action.

1.	~(D & S)	A/≡I
2.	~D v ~S	1 DeM
3.	~S v ~D	2 Comm
4.	S ⊃ ~D	3 Impl
5.	S ⊃ ~D	A/≡I
6.	~S v ~D	5 Impl
7.	~D v ~S	6 Comm
8.	~(D & S)	7 DeM
9.	~(D & S) ≡ (S ⊃ ~D)	1-4,5-8 ≡I

8. (A ≡ ~K) v (~B & L). ∴ K ⊃ ~(A & ~L)

DERIVE: K ⊃ ~(A & ~L)

Going SD purist for this one.

1.	(A ≡ ~K) v (~B & L)	A
2.	K	A/⊃I
3.	A ≡ ~K	A/vE
4	A & ~L	A/~I
5.	A	4 &E
6.	~K	3,5 ≡E
7.	K	2 R
8.	~(A & ~L)	4-7 ~I
9.	~B & L	A/vE
10.	A & ~L	A/~I
11.	L	9 &E
12.	~L	10 &E
13.	~(A & ~L)	10-12 ~I
14.	~(A & ~L)	1,3-8,9-13 vE
15.	K ⊃ ~(A & ~L)	2-14 ⊃I

Part 4. Combination. Translate the following argument into SL using the symbolization guide provided. If your symbolization is invalid, construct a <u>full</u> truth table for the argument and indicate at least one counterexample. If your symbolization is valid, demonstrate its validity by constructing a derivation. (20 pts)

9. Simon will get rich only if he marries well. Simon's mom will be very proud if he marries well. So, if Simon's mom will not be very proud, then Simon will neither get rich nor marry well. (R: Simon gets rich. W: Simon marries well. P: Simon's mom will be very proud.)

TRANSLATION: $R \supset W$. $W \supset P$. $\therefore\ {\sim}P \supset {\sim}(R \vee W)$

This argument is <u>truth-functionally valid</u>, so a derivation is called for. This proof uses some SD+ rules.

DERIVE: ${\sim}P \supset {\sim}(R \vee W)$

1.	$R \supset W$		A
2.	$W \supset P$		A
3.		${\sim}P$	A/$\supset$I
4.		${\sim}W$	2,3 MT
5.		${\sim}R$	1,4 MT
6.		${\sim}R\ \&\ {\sim}W$	4,5 &I
7.		${\sim}(R \vee W)$	6 DeM
8.	${\sim}P \supset {\sim}(R \vee W)$		3-7 $\supset$I