Equivalence Proof in SD and SD+

Phi 321 Formal Logic
Jared Bates, Hanover College

To prove that '~(A ⊃ B)' and 'A & ~B' are logically equivalent, two derivations are needed. Proofs #1 and #2 prove the equivalence using only SD rules. Proofs #3 and #4 exploit SD+ rules.

1

DERIVE: A & ~B

1.	~(A ⊃ B)	A
2.	~A	A/~E
3.	A	A/⊃I
4.	~B	A/~E
5.	A	3 R
6.	~A	2 R
7.	B	4-6 ~E
8.	A ⊃ B	3-7 ⊃I
9.	~(A ⊃ B)	1 R
10.	A	2-9 ~E
11.	B	A/~I
12.	A	A/⊃I
13.	B	11 R
14.	A ⊃ B	12-13 ⊃I
15.	~(A ⊃ B)	1 R
16.	~B	11-15 ~I
17.	A & ~B	10, 16 &I

2

DERIVE: ~(A ⊃ B)

1.	A & ~B	A
2.	A ⊃ B	A/~I
3.	A	1 &E
4.	B	2,3 ⊃E
5.	~B	1 &E
6.	~(A ⊃ B)	2-5 ~I

3

DERIVE: A & ~B

1.	~(A ⊃ B)	A
2.	~(~A v B)	1 Impl
3.	~~A & ~B	2 DeM
4.	A & ~B	3 DN

4

DERIVE: ~(A ⊃ B)

1.	A & ~B	A
2.	~~A & ~B	1 DN
3.	~(~A v B)	2 DeM
4.	~(A ⊃ B)	3 Impl